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Information transmittal, relativity and gravitation

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ABSTRACT

Special relativity considered in [Albert Einstein, Zur Elektrodynamik der bewegte Körper, Ann. Phys. 17 (1905) 891–921], and gravitation, studied in a series of papers, notably in [Albert Einstein, Zum gegenwärtigen Stände des Gravitationsproblemen, Phys. Z. 14 (1913) 1249–1262], are further analyzed regarding the principle of relativity, gravitation, and the notion of mass. The energy relation derived by Einstein from the relativistic Maxwell equations is applied to potential energy $W(x)$ of the gravitational field along the right line for which Einstein's transformations are valid. This defines the intensity $G(x) = \mathrm{d}W/\mathrm{d}x$ of the relativistic force of gravity along a right line of observation in the gravitational field. The force is proportional to the *observed* acceleration according to the formula $\varepsilon G(x) = \mu\xi_{\tau\tau} = \mu x_{tt}\beta^3$ where μ is the *inert* mass in the second Newton's law of motion and ε is the charge (mass) in the relativistic electromagnetic (gravitational) field. In everyday life, we see that all bodies visually fall under gravity (i.e. in a common gravitational field) with the same observed acceleration $\xi_{\tau\tau}$ as if having equal inert and gravitational masses: $\mu/\varepsilon = 1$, with respect to the synchronized time τ. However, if the principle of relativity extended by Einstein to the case of the uniformly accelerated rectilinear motion is valid, then this relation should also be true with respect to x_{tt}, that is, $(\mu/\varepsilon)\beta^3 = 1$, in proper time t of a still observer and of the carrying system (falling body), thus, depending on velocity v at which the acceleration $\xi_{\tau\tau}$ is measured. This means that the inert mass μ and the gravitational mass ε can be considered equal *only* at $v = 0$, and otherwise are related by the equation $\varepsilon = \mu\beta^3 \geq \mu$, where Einstein's calibration factor $\beta = [1 - (v/V)^2]^{-0.5} \geq 1$, $|v| < V$, and $\beta \cong 1$ for small $|v|$ compared with the speed of light $V = 300\,000$ km/s at which we see the falling bodies. If $v > 0$, then the *observed* gravitational mass ε is *greater* than the inert mass μ. The increase of mass is concurrent with the increase of tensions that at high velocities $v \to V$ induce overheating in the particle accelerators and colliders. To comply with the nature of observation, the information transmittal signals are incorporated in the Lorentz invariant of the 4*D* geometry, leading to the local invariants of relativistic dynamics that include gravitation and the speed of signals used in observation of moving bodies. With the same communication signals, those invariants hold for the synchronized time and coordinates of moving systems irrespective of their relative velocities. A procedure is developed for measurement and computation of the accelerations produced by variable gravitational and/or electromagnetic fields through the measurements of velocities of a moving body, so that the motion of the body and the field of forces acting on it can be fully identified. The results open new avenues for research in the theory of relativity and its applications.

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1. Introduction

In the seminal paper [1], see also [2, tome I, pp. 7–35], Albert Einstein has derived the mathematical transformations of special relativity from the natural phenomena of motion and signal propagation in their interrelation with respect to time, length and velocity. He then applied those transformations to electrodynamics (Maxwell–Hertz equations), to aberration and Doppler's effect, to the explanation of the pressure of light, and to dynamics and energy of a weakly accelerated electron (1905). Quite soon, in 1907, Einstein turned to the problem of gravitation in [3], see also [2, tome I, pp. 105–114, Principle of relativity and gravitation], and many papers followed in which Einstein was analyzing the possibility of including the gravitational field into the theory of special relativity; see the editors' remark in [2, tome I, p. 174]. This attempt to include gravitation into the mathematical framework obtained from simpler phenomena in kinematics of inertial systems encountered difficulties, and it is not completed to date, after more than a century of theoretical and experimental investigations.

In this paper, it is demonstrated that the inert and gravitational masses, equal at rest, are *not* identical in systems moving under gravitation but subject to relativistic increase due to contraction of the observed time τ. The concurrent increase of tensions in the Maxwell–Hertz equations at high velocities of accelerated particles may cause the increase of induced heat above the safety levels. Relativistic problems in particle accelerators and under gravitation require the consideration of special relativity at *variable* velocities as in [4,5]. Preservation of energy allows us to identify the gravitational field at a distance by measuring the actual accelerations of a body with the radar or other appropriate signals while computing the force of attraction and the actual motion of the body. To comply with the nature of observation, the information transmittal signals are incorporated in the Lorentz invariant of the 4D geometry, leading to the local invariants of relativistic dynamics that include gravitation and correspond to those signals in interacting physical processes. With the same communication signals, the local invariants of relativistic dynamics hold for the synchronized time and coordinates of moving systems irrespective of their relative velocities. In this way, it is possible to directly measure the combined action of a given field of forces in some direction of interest, and then reconstruct the motion in this direction under the measured intensity of the actually existing field.

The paper is organized as follows. In Section 2, Einstein's definition of simultaneity is reproduced in quotations from his basic paper [1, Sections 1–2]. Section 3 presents Einstein's coordinate transformations from [1, Sections 3,4] with a brief discussion of time and length contraction phenomena. In Section 4, relativistic representations are studied for the scalar notion of mass. In Section 5, relativistic increase of tensions and overheating in particle accelerators are discussed. In Section 6, the preservation of the energy relation for the relativistic Maxwell–Hertz equations is extended to gravitation. Section 7 contains the general notes by Einstein about the principle of relativity, the conservation laws and the problem of gravitation. In Section 8, the Lorentz invariants and transformations are reproduced and compared with the wave invariants considered by Einstein. In Section 9, a method is developed for relativistic identification of the actual field of forces in accelerated motions. In Section 10, the information transmittal signals are included in the Lorentz invariant, leading to local invariants of relativistic dynamics under gravitation. Section 11 presents concluding remarks followed by the references immediately relative to the problems considered.

2. Definition of simultaneity [1, Sections 1,2]

This is the title of the first section from which we reproduce the original Einstein's description of time and simultaneity in the English translation from the Russian edition [2, tome I, pp. 8–10]. For a coordinate system "in which are valid the equations of mechanics of Newton", called "still system", or system at rest, the following is written.

"When desired to describe a *motion* of a material point, we specify the values of its coordinates as functions of time. Thereby it should be noted that such mathematical description has physical sense only if it is first understood what is meant by "time". We should pay attention to the fact that all our considerations in which time plays a role are always the considerations about *simultaneous* events". Then we read in [2, tome I, p. 9]:

"If at point *A* of a space there is a clock, then an observer at *A* can establish the time of events in immediate proximity of *A* by observing the simultaneous with those events positions of hands of the clock. If at another point *B* of the space there is also a clock (we add "identical as the one at *A*"), then in immediate proximity of *B* it is also possible to make time estimate of events by an observer at *B*. However, it is impossible without further hypotheses to compare the timing of an event at *A* with an event at *B*; we have yet defined only "*A*-time" and "*B*-time" but not the common for A and *B* "time". The latter can be established *by introducing a definition* that "time" necessary for passing of a ray of light from *A* to *B* is equal to "time" necessary for passing of a ray of light from *B* to *A*. Consider that at a moment t_A of "*A*-time" a ray of light leaves from *A* to *B* and is reflected at a moment t_B of "*B*-time" from *B* to *A* returning back at *A* at a moment t'_A of "*A*-time". The clocks at A and *B* will be, by definition, synchronized, if

$$t_B - t_A = t'_A - t_B. \tag{1}$$

We assume that this definition of synchronization can be made in a non-contradictory manner, and furthermore, for as many points as desired, thus, the following statements are valid:

(1) if the clock at *B* is synchronized with the clock at *A*, then the clock at *A* is synchronized with the clock at *B*;

(2) if the clock at A is synchronized with the clock at B and with the clock at C, then the clocks at B and C are also synchronized with respect to each other.

Thus, using certain (thoughtful) physical experiments, we have established what should be understood as synchronized located in different places still clocks, and thereby we evidently achieved definitions of the concepts: "simultaneity" and "time". "Time" of an event means simultaneous with the event indication of a still clock which is located at the place of the event and which is synchronized with certain still clock, thereby with one and the same clock under all definitions of time.

According to experiments, we also assume that the value

$$2AB/(t'_A - t_A) = V \quad (AB \text{ is the length of a segment}) \tag{2}$$

is a universal constant (the speed of light in vacuum).

It is essential that we have defined time with the help of still clocks in a system at rest; we shall call this time that belongs to a system at rest, "the time of still system".

Further considerations are based on the principle of relativity and on the principle of constancy of the speed of light. We formulate both principles as follows.

1. Laws which govern the changes of state of physical systems do not depend on which of the two coordinate systems, moving with respect to each other with a constant speed along a right line, these changes relate.
2. Every ray of light propagates in a "still" system of coordinates with certain speed V irrespective of whether the ray of light is issued by a resting or moving source.

Thereby, formula (2) applies, and the "segment of time" should be understood in the sense of the above definition".

3. Einstein's coordinate transformations [1, Sections 3,4]

We now quote the passages from [2, tome I, pp. 13–14] related to theory of the time transformation. "Consider in a "still" space two 3D Cartesian frames with a common origin and parallel axes, each equipped with scales and clocks which are identical in both frames. Now, let the origin of one of those frames (k) be in motion with a constant speed v in the direction of increasing x of the other frame (K) which is at rest. Then, to each moment t of still frame (K) corresponds a certain position of axes of moving frame (k) whose axes can be assumed parallel to the axes of the still frame (K).

Let the space in the still frame (K) be graduated with its scale at rest, and same for the space in the moving frame (k) graduated with its scale, at rest with respect to (k), yielding coordinates x, y, z in (K) and ξ, η, ζ in (k). Using light signals as described in [1, Section 1], see above, let us define time t in (K) and τ in (k) with the clocks at rest in each frame.

In this way, to the values x, y, z, t which define the place and time of an event in the still frame (K), there will correspond the values ξ, η, ζ, τ that define the same event in the moving frame (k), and we have to find the system of equations that link those values of coordinates and times.

First of all, it is clear that those equations must be *linear* according to the property of homogeneity which we ascribe to the space and time.

If we denote $x' = x - vt$, then it is clear that to a point at rest in the system (k) will correspond certain, independent of time values x', y, z. Let us determine τ as a function of x', y, z, t, which would mean that τ corresponds to the readings of clocks at rest in the moving frame (k) synchronized with the clocks in the still frame (K) by the rule (1)".

Choosing in (1) the point A as the origin of the moving frame (k) and sending at the moment $\tau_0 = t_A$ a ray of light along the X-axis to the point x' (point B) which is reflected back at the moment $\tau_1 = t_B$ to the origin where it comes at the moment $\tau_2 = t'_A$, we have from (1) the following equation: $\tau_1 - \tau_0 = \tau_2 - \tau_1$ which is written in [1, Section 3], quote from [2, tome I, p. 14, the first equation], in the form:

$$\text{“}0.5(\tau_0 + \tau_2) = \tau_1, \tag{3}$$

or, specifying the arguments of the function τ and using the principle of constancy of the speed of light in the system at rest (K), we have

$$0.5[\tau_0\,(0,0,0,t) + \tau_2(0,0,0,\{t + x'/(V-v) + x'/(V+v)\})] = \tau_1[x', 0, 0, t + x'/(V-v)]. \tag{4}$$

If x' is taken infinitesimally small, then it follows that

$$0.5[1/(V-v) + 1/(V+v)]\partial\tau/\partial t = \partial\tau/\partial x' + [1/(V-v)]\partial\tau/\partial t, \tag{5}$$

or

$$\partial\tau/\partial x' + [v/(V^2 - v^2)]\partial\tau/\partial t = 0. \tag{6}$$

It must be noted that we could take, instead of the origin, any other point to send a ray of light, therefore, the last equation is valid for all values x', y, z.

Since the light along the axes Y and Z, if observed from the system at rest, always propagates with the velocity $(V^2 - v^2)^{0.5}$, the similar argument applied to these axes yields $\partial\tau/\partial y = 0$, $\partial\tau/\partial z = 0$. Since τ is a *linear* function, from these equations it follows that

$$\tau = a[t - vx'/(V^2 - v^2)], \tag{7}$$

where $a = \varphi(v)$ is an yet unknown function, and for brevity it is taken that at the origin of the moving frame (k) if $\tau = 0$, so also $t = 0$. (Einstein's notations; see [2, I, pp. 14–15].)

For more than a century, time and again, different reservations and/or doubts appeared in the literature as to the validity and precision of the classical relativity theory. To dispel any doubt and to make special relativity understandable to everybody, we assume the constancy of V and v, $|v| < V$, and Einstein's synchronization method (3)–(4) based on the rays of light, and try to find a linear function with undetermined coefficients

$$\tau(x', y, z, t) = at + bx', \quad a, b = \text{const}, \tag{8}$$

that would satisfy Eq. (4) *identically* with respect to t and x'. Substituting (8) into (4) and noting that $y = z \equiv 0$ in (4), for a ray of light along the X-axis, we have

$$0.5[at + a\{t + x'/(V - v) + x'/(V + v)\}] \equiv bx' + a[t + x'/(V - v)], \quad \forall t, \forall x'. \tag{9}$$

Multiplying (9) by 2 and canceling the terms with at on both sides, we get

$$a[x'/(V - v) + x'/(V + v)] \equiv 2x'[b + a/(V - v)], \quad \forall x'. \tag{10}$$

Simplifying (10), without division by x', we see that the identity holds if and only if the constants a and b are chosen from the equation

$$aV/(V^2 - v^2) = b + a/(V - v), \quad |v| < V, \tag{11}$$

that is,

$$b = aV/(V^2 - v^2) - a/(V - v) = -av/(V^2 - v^2), \tag{12}$$

yielding in (8)

$$\tau(x', y, z, t) = a[t - vx'/(V^2 - v^2)], \quad |v| < V, \tag{13}$$

which coincides with (7). We see that a linear homogeneous time transformation (13) corresponding to the synchronization equations (3)–(4) exists for all t, x', $|v| < V$, with arbitrary nonzero calibrating factor $a(.)$ to be determined by additional requirements.

Substituting $x' = x - vt$ into (13) yields

$$\tau = a[t - v(x - vt)/(V^2 - v^2)] = a\alpha^2(t - vx/V^2), \quad \alpha^2 = V^2/(V^2 - v^2), \tag{14}$$

so that the time τ is really homogeneous in t, x' of (13) and in t, x of (14). According to initial conditions, a constant may be added in (8), thus, to (7) and (14), as noted by Einstein [2, I, p. 16], which constant is canceled after the substitution of (8) into (3), (4).

The analogue of this case is obtained for the Y-axis and Z-axis with rays of light along those axes propagating with velocity $w = (V^2 - v^2)^{0.5}$, if observed from the system at rest, the same for direct and reflected rays. After simple calculation (for details, see [6, pp. 1561–1562]), one can see that model (8) is valid for all the three axes, thus the linear homogeneous transformations (13) and (14) not depending on y, z are universal for all the three axes X, Y, Z in (K).

The factor $a(.)$ has been determined by Einstein [1] or [2, I, pp. 16–17] by introducing "…one more, the third coordinate system (K'), which with respect to system (k) is in translational motion parallel to ξ-axis in such a way that its origin moves with velocity $-v$ along ξ-axis". Such a choice of (K') implies "that transformation from (K) into (K') must be the identity transformation". [2, I, p. 17] Omitting details of derivation which can be found in [6, Section 7, pp. 1563–1564], this yields relativistic transformations [1, Section 3] well known in the literature:

$$\tau = \beta(t - vx/V^2), \qquad \xi = \beta(x - vt), \qquad \eta = y, \qquad \zeta = z, \qquad \beta = [1 - (v/V)^2]^{-0.5} \geq 1, \tag{15}$$

where β is the calibration factor corresponding to (1), (3), (7), (14). Since $\alpha^2 = \beta^2$ in (14), $a = \beta^{-1}$ in (7), (13), (14). Note that (15) are invertible with determinant $\Delta = 1$, for the first two equations, if $0 < v < V$. For $v \in [0, V)$ we have $\beta \in [1, \infty)$ monotonically increasing with v. If (K) is observed from the moving frame (k), then one has to invert (15) and replace v for $-v$ with which (K) moves with respect to (k) if (k) is considered "at rest", yielding $t = \beta(\tau - v\xi/V^2)$, $x = \beta(\xi - v\tau)$, same as in (15). If $\xi = v\tau$, then observer in (k) "sees" $x = 0$, at rest, but $t = \beta\tau(1 - v^2/V^2) = \tau\beta^{-1} < \tau$, contraction of time in (K) if observed from (k).

The relativistic contraction of time is experimentally confirmed by the discovery of μ-mesons at the sea level. These are particles born in cosmic rays that have a short lifetime about 2 microseconds (in observed τ-time). They are moving with velocity that equals 99.5% of the speed of light which amounts to $v = 2.985 \times 10^{10}$ cm/s $= 2.985 \times 10^8$ m/s. With this velocity and lifetime of $\tau^0 = 2 \times 10^{-6}$ s, these particles could enter the atmosphere not deeper than at $l = v\tau^0 \cong 600$ m. However, the observed τ^0-lifetime actually represents the contracted natural lifetime $t^0 = \beta\tau^0 = (1 - v^2/V^2)^{-0.5}\tau^0 = (1 - 0.990)^{-0.5}\tau^0 = 10\tau^0$, during which the particles would enter the atmosphere at $l^0 = vt^0 = 10v\tau^0 = 6000$ m that corresponds to the sea level at which the μ-mesons have been discovered. It means that they exist not by our observations within the span of τ^0-lifetime, but by their own nature within their natural t^0-lifetime.

If we observe a process (clock) unfolding in a moving frame, using rays of light or radar, the unit of time Δt in the motion of that process seems shorter, $\Delta\tau = \beta^{-1}\Delta t < \Delta t$. It is instructive that contraction of time happens in exactly the same proportion $\beta^{-1} < 1$ as contraction of the size of a solid in the direction of the velocity v of a moving frame; see [1, Section 4]; [2, I, p. 18]. It proves the perfect similarity in contraction of time and the relativistic coordinate observed along the right line of velocity, in accordance with assumption (2).

Remark 3.1. Note that τ, ξ, η, ζ are the *observed* time and coordinates in which real processes evolving in (k) are *distorted* when observed from (K); see [6, Sec. 8]. It means that times τ and t are *not* the same but present *different* time entities whereby τ is the *image* of t if observed from (K) and, according to the principle of relativity, Law 1 in Section 2, t is the *proper* time in (K) and in (k) if observed from the same system.

4. The scalar notion of mass and its relativistic representations

In Section 10 of [1] entitled "Dynamics of weakly accelerated electron" Albert Einstein writes (translation from [2, tome I, pp. 32–34], notations and format by Einstein):

"Suppose that in electromagnetic field a point-wise particle is moving with electrical charge ε (called "electron" in what follows), and about the law of its motion we shall assume only the following.

If an electron is at rest during certain interval of time, then at immediately following time moment the motion of the electron, since it is slow, will be described by equations:

$$\mu \mathrm{d}^2 x/\mathrm{d}t^2 = \varepsilon X, \qquad \mu \mathrm{d}^2 y/\mathrm{d}t^2 = \varepsilon Y, \qquad \mu \mathrm{d}^2 z/\mathrm{d}t^2 = \varepsilon Z, \tag{16}$$

where x, y, z are coordinates of the electron, and μ is the mass of the electron.

Further, suppose that the electron during certain interval of time has velocity v. Let us find a law according to which the electron is moving at immediately following thereafter time moment.

Without loss of generality, we can assume, and we assume indeed, that at that moment, when we begin observation, our electron is at the origin and is moving along the X-axis of system (K) with velocity v. In this case, it is clear that at that moment of time $(t = 0)$ the electron is at rest with respect to coordinate system (k) moving parallel to the X-axis with constant velocity v.

From the above assumption combined with the principle of relativity, it follows that equations of motion of the electron observed from system (k) during time immediately following after $t = 0$ (at small values of t), have the form:

$$\mu \mathrm{d}^2 \xi/\mathrm{d}\tau^2 = \varepsilon X', \qquad \mu \mathrm{d}^2 \eta/\mathrm{d}\tau^2 = \varepsilon Y', \qquad \mu \mathrm{d}^2 \zeta/\mathrm{d}\tau^2 = \varepsilon Z', \tag{17}$$

where denoted by $\xi, \eta, \zeta, \tau, X', Y', Z'$ are values related to system (k). If we also set that for $t = x = y = z = 0$ we have $\tau = \xi = \eta = \zeta = 0$, then the formulae of transformation from Sections 3 and 6 will be valid, and thus, the following equations will hold:

$$\begin{aligned}
&\tau = \beta(t - vx/V^2),\\
&\xi = \beta(x - vt), \qquad X' = X,\\
&\eta = y, \qquad Y' = \beta(Y - vN/V),\\
&\zeta = z, \qquad Z' = \beta(Z + vM/V).
\end{aligned} \tag{18}$$

Making use of these equations, we transform Eqs. (17) from system (k) to system (K), yielding

$$\begin{aligned}
&\mathrm{d}^2x/\mathrm{d}t^2 = \varepsilon\mu^{-1}\beta^{-3}X,\\
&\mathrm{d}^2y/\mathrm{d}t^2 = \varepsilon\mu^{-1}\beta^{-1}(Y - vN/V), \qquad (A)\\
&\mathrm{d}^2z/\mathrm{d}t^2 = \varepsilon\mu^{-1}\beta^{-1}(Z + vM/V).
\end{aligned} \tag{19}$$

Using the usual course of argumentation, let us define now the "longitudinal" and "transverse" mass of a moving electron. Let us write equations (A) in the following form:

$$\begin{aligned}
&\mu\beta^3\mathrm{d}^2 x/\mathrm{d}t^2 = \varepsilon X = \varepsilon X',\\
&\mu\beta^2\mathrm{d}^2 y/\mathrm{d}t^2 = \varepsilon\beta(Y - vN/V) = \varepsilon Y',\\
&\mu\beta^2\mathrm{d}^2 z/\mathrm{d}t^2 = \varepsilon\beta(Z + vM/V) = \varepsilon Z'.
\end{aligned} \tag{20}$$

Now, we note, first of all, that $\varepsilon X', \varepsilon Y', \varepsilon Z'$ are components of electromagnetic force acting upon the electron, whereby those components are considered in the coordinate system which at a given moment is moving together with the electron with the same, as for the electron, velocity. (This force could be measured, for example, by a spring scale at rest in that system.) If now we shall call this force simply "a force acting upon the electron", and preserve the equation (for numeric values)

Mass $\times$ Acceleration $=$ Force,

and if we further define that accelerations must be measured in the still system (K), then from the above equations we obtain:

$$\begin{aligned}&\text{longitudinal mass} = \mu[1-(v/V)^2]^{-1.5} = \mu\beta^3,\\&\text{transverse mass} = \mu[1-(v/V)^2]^{-1} = \mu\beta^2.\end{aligned} \tag{21}$$

Of course, we shall get different values for masses under different definitions of forces and accelerations; thus, it is clear that in comparison of different theories of motion of an electron, one should be very careful. We note that these results about the mass are valid also for neutral material points since such a point can be treated as electron (in our sense) by adjoining an arbitrarily small electrical charge".

Remark 4.1. Considering "neutral material points" and "different values for masses under different definitions of forces and accelerations", an interesting conclusion can be made from (21). Indeed, at $v = 0$, $\beta = 1$, and the value μ presents a scalar *static* mass of a point in the still system (K). When moving with (k) along the X-axis of (K) at $v > 0$, the same mass observed in (K) from (k) and *not* moving in transverse direction with respect to the X-axis of (K) has the value $m_0 = \mu\beta^2$, being at *relative rest* with respect to $0x \in (K)$, thus called *rest mass* m_0. The same mass μ in motion at $v > 0$ along $0x \in (K)$ presents the *observed moving and increased* mass (called *longitudinal* by Einstein)

$$m = \mu\beta^3 = \beta m_0 = m_0[1-(v/V)^2]^{-0.5} = m_0 + 0.5m_0v^2/V^2 + 0.375m_0v^4/V^4 + \cdots. \tag{22}$$

It is still widely discussed in the literature, generating profound interest and curiosity; see [7]; or [8, pp. 46–47]; [9, Ch. IX, Sec. 5]; [10, p. 643]. Of course, the values $m_0 = \mu\beta^2$, $m = \beta m_0$ are the *relativistic expressions* with regard to the *inert* mass μ (at rest) observed in different motions ($m_0 = \mu\beta^2$, $m = \mu\beta^3 = \beta m_0$).

Alternative consideration. The double value for the mass in (21) was defined in order to preserve the same form of Newton's second law of motion in (16), (17) for the *proper* x, y, z, t coordinates in both (K) and (k) frames (according to the principle of relativity), and for *transformed* coordinates ξ, η, ζ, τ as *observed* in (K) from (k), whereby the *observed* forces (X', Y', Z') in (k) and original forces (X, Y, Z) in (K) are *different*, according to (18). The preservation of the mathematical representation (equations) of physical laws in transformed coordinates is *not* included in the principle of relativity as formulated by Einstein in Section 2, Law 1. Indeed, intuitively it seems clear that the laws of nature stay unchanged in all *inertial* systems at rest and systems moving with constant velocities, with respect to the *proper* time and coordinates of those inertial systems (frames). Transformations (15), however, are based on a signal propagating with some finite speed, and it is questionable that the same laws might be expressed by the same formulae in the *observed* (transformed) coordinates ξ, η, ζ, τ conditioned on the relative speed v and on the speed V of the signal propagation and quite different from the *proper* coordinates x, y, z, t, cf. Remark 3.1. Also, the notion of *mass* is *not* associated with directions or coordinate transformations, and it would be expedient to preserve the mass as a scalar characteristic of a body.

Let us demonstrate that, in fact, it is the change in *observed* accelerations to which the difference of values in (21) can be attributed, without any deviation from the principle of relativity and from the substance of the second law of motion as formulated by Newton. Considering derivatives of *observed* coordinates with respect to *observed* time in (17) and comparing them with derivatives of the *proper* coordinates with respect to the *proper* time in (16), we have, due to (15):

$$\mathrm{d}\xi/\mathrm{d}\tau = \xi_\tau = (\mathrm{d}\xi/\mathrm{d}t)/(\mathrm{d}\tau/\mathrm{d}t) = (x_t - v)/(1 - vx_t/V^2), \quad x_t = w(t) \tag{23}$$

$$\mathrm{d}^2\xi/\mathrm{d}\tau^2 = \xi_{\tau\tau} = [x_{t\tau}(1 - vx_t/V^2) + (x_t - v)vx_{t\tau}/V^2]/(1 - vx_t/V^2)^2. \tag{24}$$

One has to note that in (15) the point $(x, y, z) \in (K)$ is at rest in (K), however, in (16) that same point $(x, y, z) \in (K)$, the electron, is moving, and accelerating to velocity v, same as the constant velocity v of (k) along the $0x$-axis of (K); see Einstein's explanation after (16) which are Newtonian equations in *proper* time and coordinates of (K). This means that transformation (15) is being done in its *continuous superposition*, with accelerating point $(x, y, z) \in (K)$. In order not to confuse the constant velocity v of (k) with respect to (K) which enters (15), (23), (24) and the velocity v of the accelerating electron (same notation for *different* entities) we denoted the latter by $x_t = w(t)$, as indicated in (23). At some moment, it happens that $w(t) = v$, and at this moment the electron is at rest with respect to (k). For this reason, coordinate $x(t)$ of the electron can be differentiated in (15) as is done for x in (23)–(24), since it varies with the moving electron. Thus, we have at the moment that velocity $v = \mathrm{d}x/\mathrm{d}t = x_t$ is achieved by the electron:

$$x_{t\tau} = d(\mathrm{d}x/\mathrm{d}t)/\mathrm{d}\tau = x_{tt}/(\mathrm{d}\tau/\mathrm{d}t) = x_{tt}/\beta(1 - vx_t/V^2) = x_{tt}/\beta(1 - v^2/V^2) = x_{tt}\beta. \tag{25}$$

Continuing (24) and using (25) at the moment when $x_t = v$, we obtain

$$\xi_{\tau\tau} = x_{t\tau}\beta^{-2}\beta^4 = x_{t\tau}\beta^2 = x_{tt}\beta^3. \tag{26}$$

Repeating (23)–(26) for coordinates η, ζ, we have at that same moment $x_t = v$

$$\mathrm{d}\eta/\mathrm{d}\tau = \eta_\tau = (\mathrm{d}\eta/\mathrm{d}t)/(\mathrm{d}\tau/\mathrm{d}t) = (\mathrm{d}y/\mathrm{d}t)/\beta(1 - vx_t/V^2) = y_t\beta, \tag{27}$$

$$\mathrm{d}^2\eta/\mathrm{d}\tau^2 = \eta_{\tau\tau} = (\mathrm{d}\eta_\tau/\mathrm{d}t)/(\mathrm{d}\tau/\mathrm{d}t) = y_{tt}\beta/\beta(1 - vx_t/V^2) = y_{tt}\beta^2, \tag{28}$$

and in the same way we obtain $d^2\zeta/d\tau^2 = z_{tt}\beta^2$. Comparing these values with values in (20), (21), we see that there is no need to attribute the factors β^3, β^2 to the mass in (21) since they naturally occur in the *observed* accelerations, cf. (20), (26) and (28). This does not contradict the universally accepted formula (22). Indeed, if we prefer to deal with the Newtonian process Eqs. (17) in *all relativistic* coordinates ξ, η, ζ, τ observed in (K) from (k), then we have to consider the static scalar mass $\mu = \text{const}$ of the electron with the *observed* accelerations in (17), cf. with (24), (26), (28). However, if we prefer to deal with the process Eqs. (19) or (20) in the *proper* coordinates x, y, z, t of (K) and (k), which are *not* Newtonian equations of motion, then, in order to *write* them as Newtonian equations, we have to consider *different* masses in (20)–(22) with proper accelerations as in (20). The process is the same, but the mathematical representations are different. If we accept (17) with $\mu = \text{const}$, then the *proper* acceleration $x_{tt} = \xi_{\tau\tau}\beta^{-3} < \xi_{\tau\tau}$ with the same effect as in (20) where acceleration x_{tt} is less for the same force $\varepsilon X'$, due to the greater mass $m = \mu\beta^3$ in (20) and (22). The coordinates can be chosen at will, but the result remains the same: the increase of the mass in (22) reflects the smaller (than observed $\xi_{\tau\tau}$) actual acceleration x_{tt} in the moving system (k). The principle of relativity implies *duality* of the mathematical representations.

Remark 4.2. Of course, all formulae (20)–(28) are different if $dx/dt = x_t = w(t) \neq v$ at some moment t indicated above. Also, for a spacecraft driven by reactive forces or an accelerating particle at high velocity with increasing mass, the representation of the second Newton's law of motion is modified [11, Sec. 6], and the observed mass is distorted by relativistic transformations as well as all parameters and physical laws that depend on velocity directly or indirectly.

Remark 4.3. Regarding Einstein's consideration of longitudinal and transverse masses, see (21), together with definition that "The mass of a body is a measure of energy contained in it" [7, p. 641]; or [2, tome I, p. 38], and comparing it with the well known equation $E = mc^2$, it is clear that treating the mass as a vector means treating the energy in the same way since c^2 is a scalar. We can see this in (29), (30), Section 6, where electrostatic or gravitational field energy is transformed into kinetic energy of a body (electron). Albeit the consideration of energy as a vector does not comply with current views, the problem is not so simple. Kinetic energy $E_k = 0.5mv^2$ is clearly directed along the velocity vector v, and along this direction it can be used to produce electricity in hydroelectric plants. The same directional effects can be observed in gyroscopic systems. Thus, retaining the notion of mass as a scalar, as postulated by Newton [12]: "The change of motion is proportional to the motive force impressed and is made in the direction of the right line in which the force is impressed", and respecting Einstein's relation $E = mc^2$, we have to emphasize that this energy E is the energy of electromagnetic waves which propagate as *spherical* waves with the *speed of light* as specified by Einstein in [1,13]; see also [14, p. 2504].

5. Relativistic breakdown and safety in particle accelerators

Einstein's relativistic transformations of the Maxwell–Hertz equations for vacuum (see [1, Sections 6,10] or [14, pp. 2494–2496]), presented in part by (18) in Section 4, are closely related to the safety of particle accelerators and colliders. For simplicity, let us consider first a trivial example of a short circuit. Ohm's law for a segment of the circuit states that the voltage U across the circuit equals the product of the resistance R in ohms by the current I in amperes: $U = RI$. The amount of heat Q (in calories) generated in a conducting segment equals $Q = 0.24IUt$ for I in amperes, U in volts and t in seconds. Since $I = U/R$, so we have $Q = 0.24U^2t/R$. Short circuit occurs if $R \to 0$, in which case the heat $Q \to \infty$. If one puts a piece of wire into an electric outlet, the fuse will be burnt at once. The same would happen if $R \geq R_0 > 0$ but the voltage $U \to \infty$. In a household it is impossible since normally $U = 110$ V, or 220 V, depending on a country.

However, in particle accelerators and colliders, the large increase of electrical tension U^* is quite possible, especially in colliders in which charged particles (protons, ions) are accelerated to very high velocities v. Indeed, the normal electric Y, Z and magnetic N, M field tensions in a collider when applied to a stream of charged particles are multiplied by the factor $\beta = [1 - (v/V)^2]^{-0.5}$ from (15) producing the *effective* electric tensions Y', Z', see (18) and (20), that accelerate the charged particles to very high velocities v approaching the speed of light V in (15), so that $\beta \to \infty$, resulting in $Y' \to \infty$, $Z' \to \infty$ in (18) too. The same happens with magnetic tensions M', N'; see [14, p. 2494]. This effect may produce relativistic breakdown, "a short circuit", when $Q \to \infty$ as $U^* \to \infty$.

The rate at which the tensions may increase in modern accelerators and colliders can be counted with data of the CERN Large Hadron Collider where "it is intended to collide opposing beams of protons or lead ions, each moving at approximately 99.999999% of the speed of light". [http://en.wikipedia.org/wiki/Large_Hadron_Collider (Update 2008-11-19, p. 1)]. At *this* speed of protons, we have $\beta^* = 7071$; it means that normal tensions must increase by a factor 7071 to produce such a velocity of moving protons, which would increase the tension U^* by β^*, and heat by $\beta^{*2} = 50\,000\,031 \cong 10^{7.7}$ times. It is no surprise that an accident may occur at such increase of heat. Such relativistic breakdown (a "short circuit") is not necessarily a disaster. Since it normally occurs *before* the explosion, it may cause a local disturbance, preventing a major catastrophe.

6. Preservation of energy means the relativity of gravitation

After the note about masses in (21), Einstein continues: "Let us determine the kinetic energy of an electron. If an electron is moving from the origin of system (K) with initial velocity 0 along the X-axis under the action of electrostatic force X, then it is clear that the energy taken from the electrostatic field is equal $\int \varepsilon X dx$. Since the electron is accelerating slowly and because of that does not have to give away energy in the form of radiation, the energy taken from the electrostatic field must be set equal to the energy of motion W of the electron. Taking into account that during the entire process of motion the first of equations (A) is valid, we obtain:

$$W = \int \varepsilon X \mathrm{d}x = \int \mu\beta^3 v \mathrm{d}v = \mu \int_0^v v[1-(v/V)^2]^{-3/2}\,\mathrm{d}v = \mu V^2\{[1-(v/V)^2]^{-0.5} - 1\}. \tag{29}$$

For $v = V$ the value W becomes, thus, infinitely large. As in previous results and here as well, velocities greater than the speed of light cannot exist. This expression for kinetic energy must be valid also for any masses due to the above mentioned argument".

N.B. The integral in (29) can be taken by substitution $v/V = \sin\varphi$, thus, $\beta = \sec\varphi$, and the value at the right-hand side can be written simply as $\mu V^2\,(\beta - 1) = \mu V^2(\sec\varphi - 1)$.

The energy conservation law embodied in Eq. (29), if written with indication of all limits in the integrals, takes the form:

$$W = \int_{x1}^{x2} \varepsilon X \mathrm{d}x = \int_0^v \mu\beta^3 v \mathrm{d}v = \mu \int_0^v v[1-(v/V)^2]^{-3/2}\,\mathrm{d}v = \mu V^2\{[1-(v/V)^2]^{-0.5} - 1\}. \tag{30}$$

With $x1$ and $x2$ being constant and such that $x_t = 0$ at $x1$, $x_t = w(t) = v$ at $x2$, the first integral does not depend on V nor on constant relative velocity v of (k), whereas the second and third integrals in (30) depend on V and $v = x_t$. It means that one and the same "energy taken from electrostatic field" takes different numerical values if observed at different velocities V and v (by different signals at different relative speeds). With respect to the energy conservation law, this fact can be explained only as *scaling* by the signals and instruments produced by observation in the presence of relativistic effects.

It is clear that the energy conservation equations (29) and (30) do not depend on the physical nature of the field from which the energy is transformed into the energy of motion. If we consider a neutral material point in place of the electron, as noted above by Einstein, and take $X := G$ as intensity of the force of gravity along the right line between the material point and an attracting body (e.g., Earth), instead of being the electric tension acting on a charge ε, then (29) and (30) will be exactly the same and the energy in (30) will represent the potential energy of the gravitational mass ε. Now, using the model of Einstein with variable $x_2 = x(t)$, $x_t = \mathrm{d}x/\mathrm{d}t = w(t) = v$ at $x2$, as in (30), we get the force of attraction per unit mass, intensity $G(x)$, as derivative of the potential energy $W(x)$:

$$\varepsilon G(x) = \varepsilon X(x) = \varepsilon \mathrm{d}W/\mathrm{d}x = \mu V^2\,\mathrm{d}\beta/\mathrm{d}x = \mu\beta^3\,v\mathrm{d}v/\mathrm{d}x = \mu\beta^3\,\mathrm{d}v/\mathrm{d}t = \mu x_{tt}\,\beta^3 = \mu\xi_{\tau\tau}, \tag{31}$$

for the same observed acceleration $\xi_{\tau\tau}$ as in (26). From (31), one can see the following:

1. Formula (31) for the intensity of gravitation follows from the change in energy W according to Eqs. (29)–(30) for a unit mass accelerating in gravitational field, or for a charged point (electron) accelerating in electromagnetic field, both governed by the same equations. If $v = \text{const}$ (inertial system), then $G(x) \equiv 0$. Since in reality $G(x) \neq 0$, so $v \neq \text{const}$, and Einstein's transformations (15) should be upgraded for this case.

2. The right-hand side of (30) and $\xi_{\tau\tau}$ in (31) tend to infinity as $v \to V$, so the *observed* kinetic energy tends to infinity, with the rest mass μ and "energy taken from electrostatic field" (the first integral) remaining constant. Respecting the energy conservation law, it means that relativistic transformations produce distortion of the image in observation as argued earlier in [6]. In this case, the observed time $\tau \to \infty$, which means that the observation cannot be accomplished in finite time, thus the electron in motion at velocities close to the speed of light becomes undetectable by signals propagating at same velocities. It means that gravitation in such cases also cannot be measured.

3. The assumption of equality of the gravitational and inert masses $\varepsilon = \mu$ is not required. Indeed, if $\varepsilon = \mu\beta^3$, then the weight $P = \varepsilon G(x) = \varepsilon x_{tt} = \mu\beta^3 x_{tt} = mg$, cf. Remark 4.1, in the proper time and coordinates as accepted in everyday life. If $v = 0$, then $\beta = 1$ and $\varepsilon = \mu$. If $v \to V$, then $\beta \to \infty$, and with $g = x_{tt}$ we have $m = \varepsilon = \mu\beta^3 \to \infty$ as in (22), so that, for a *finite* intensity $G(x)$ of the gravitational field, the *observed* mass ε and the relativistic, thus, *distorted* force of attraction (weight) tend to infinity: $\varepsilon \to \infty$, and also $P = \varepsilon G(x) \to \infty$, an illusory effect sometimes called a black hole.

7. Principle of relativity, accelerated systems and gravitation

Here, we reproduce some notes by Einstein which are important for the understanding of the problem of gravitation in its historical perspective (notations and italics by Einstein).

In his first paper that contains some remarks on gravitation, Albert Einstein writes [3, Ch. 5, Principle of relativity and gravitation], translated from [2, tome I, pp. 105–106]:

I. "Up to date, we applied the principle of relativity, i.e. requirement of independence of the laws of nature from the state of motion of a coordinate system, only to *non-accelerated* coordinate systems. Can one accept that the principle of relativity holds also for systems moving with acceleration with respect to each other?

Consider two coordinate systems Σ_1 and Σ_2. Let Σ_1 move with acceleration in the direction of its axis X, and let its acceleration (constant in time) be equal to γ. Suppose that Σ_2 is at rest but remains in a uniform gravitational field which gives to all bodies the acceleration $-\gamma$ in the direction of the axis X. As is well known, the physical laws with respect to Σ_1 do not differ from laws respecting Σ_2; it is due to the fact that in a gravitational field all bodies are accelerated equally. Therefore, under the current state of our knowledge, there are no grounds to believe that systems Σ_1 and Σ_2 in some kind are different from each other, and in what follows we assume the entire physical equivalence of the gravitational field and the corresponding acceleration of a coordinate system. This assumption extends the principle of relativity to the case of the uniformly accelerated rectilinear motion of a coordinate system. The heuristic value of this assumption is in that it allows us to replace a homogeneous field of gravity with the uniformly accelerated coordinate system which, up to certain degree, admits theoretical consideration''.

Remark 7.1. Over infinitesimally small intervals of time, non-uniform (variable) accelerations can be approximated by constant vectors up to any precision, as required above. This assumption concerns the *effective forces* in the sense of [11, pp. 1281–1282] that may be caused by relativistic increase of the mass at high velocities or other reactive forces. It can be used to replace unknown gravitational and electromagnetic fields by the field of explicit constant accelerations along a discretized piece-wise linear trajectory, if they are postulated or measured at a distance; see Section 9. The supposition of "a uniform gravitational field" is good for clarity. In reality, it is needless and contradicts the known laws of central attraction where the force of attraction is inversely proportional to the square of the distance (Newton, Weber). Piece-wise linear constant accelerations along a discretized piece-wise linear trajectory are in agreement with those laws, containing the uniformity assumption within small intervals of time, which provides for the possibility of identification of the unknown field of forces, as described in Section 9.

II. In [15] Albert Einstein writes (translation from [2, tome I, pp. 273–276]): "The common attraction of masses belongs to that area of physical phenomena which was the first to get theoretical consideration. The laws of gravitation and motion of space bodies were turned by Newton to a simple law of motion of a material point and to the law of interaction of two mutually attracted material points. These laws happened to be so precise that from the experimental point of view there are no specific reasons to doubt their strict applicability. If, despite all that, there is hardly a physicist at present time who would trust in strict validity of these laws, this should be related to the …changing influence of our knowledge about electromagnetic phenomena for the past decades.

Before Maxwell, electromagnetic phenomena were reduced to elementary laws which were constructed as precise as possible to the model of Newton's law of gravitation. According to those laws, the interaction of electric charges, magnetic masses, elementary currents, etc., has the mode of far-action which does not need any time for its propagation in space. Then, 25 years ago, H. Hertz in his ingenious experimental investigation about the propagation of the electromagnetic field has shown that for the propagation of electric actions the time is required. Thereby, he has helped to assure the victory of Maxwell's theory in which, instead of direct far-action, partial differential equations are used. After the time when the invalidity of the theory of far-action was proved in the area of electrodynamics, the trust in the correctness of Newtonian theory of far-action has also been shaken. It should have given way to conviction that Newton's law of gravitation gives the same incomplete description for the multitude of gravitational phenomena as Coulomb's laws for electrostatics and magnetostatics described in the electromagnetic phenomena. The fact that up to date the Newtonian law happened to be sufficient for calculation of the motion of space bodies should be attributed to small velocities and accelerations in this motion …. Although the faith in the overall significance of the Newtonian law of far-action was thus shaken, direct reasons for generalization of the theory of Newton were absent. However, for those who are convinced in the correctness of the theory of relativity, such direct reason today exists. Indeed, according to the theory of relativity, there are no means in nature permitting us to send signals at a superluminal velocity. On the other hand, it is obvious that in the case of strict satisfaction of the law of Newton, we could apply gravitation for instantaneous transmission of signals from the area A to a distant area B, since the motion of gravitating mass in A should have, as a consequence, the simultaneous changes of the gravitational field in B, in contradiction with the theory of relativity.

But the theory of relativity not only suggests to modify the theory of Newton; fortunately, it also significantly restricts the possibility of its modification. This being absent, the generalization of the theory of Newton would be hopeless…we know only the interactions between masses at rest, thereby, possibly, only in the first approximation.

Diversity of possible generalizations is restricted by the theory of relativity since, in accordance with it, the time coordinate, up to difference in sign, enters all systems of equations in the same way as the three space coordinates. This, not quite precisely formulated here, deep formal rule of Minkowski, has, as it appeared, much importance in its role as auxiliary means for finding the corresponding equations from the theory of relativity.

Sec. 2. The simplest physical hypotheses about gravitational field

Below, we indicate some general postulates which *can* be accepted (*not necessarily* all) in the theory of gravitation.

1. Satisfaction of the laws of preservation of the impulse and energy.
2. Equality of the inert and gravitational masses of closed systems.
3. Satisfaction of the theory of relativity (in a more specific sense), i.e. equations must be covariant with respect to linear orthogonal substitutions (generalized Lorentz transformations).
4. The observed laws of nature must not depend on absolute values of gravitational potential(s). Physically, this means the following: the multitude of constraints between observed values that can be found in some laboratory should not change if the whole laboratory is moved in an area with a different gravitational potential (constant in space and time).

Let us make the following remarks about these postulates. All theoreticians agree that postulate 1 should be satisfied. Not so common is the conviction that it is necessary to comply with postulate 3. For example, M. Abraham advanced a theory of gravitation in which postulate 3 is not satisfied .... In our opinion, surely it is necessary to adhere to postulate 3, if the converse is not proven; as soon as we reject this postulate, diversity of possibilities becomes boundless. For more precise consideration, postulate 2 is necessary which should be adhered to if the converse is not proven. This postulate is supported first of all by experimental facts that all bodies in the gravitational field are falling with the same acceleration.... According to the theory of relativity, the inert mass of a closed system (the system is considered as a whole) is defined by its energy. Due to the postulate 2, the same should be true for the *gravitational* mass. Hence, if a state of a system is changing arbitrarily, but so that its full energy does not change, then the gravitational influence of the system is not changing, even if a part of the energy of the system is transformed into the gravitational energy. At last, postulate 4, probably, cannot be confirmed by experiments. It is justified by nothing else but the faith in simplicity of the laws of nature, and we cannot be sure that it is satisfied in the same right as in the case of the other three mentioned axioms.

We are well aware that postulates 2–4 resemble rather a scientific symbol of faith than a reliable foundation. We are also far from stating that both generalizations described below of the theory of Newton are uniquely possible; however, I still have the courage to say that under the current state of our knowledge they are the most *natural*".

(In the sections of [15] that follow, Einstein writes about the theory of Nordström and a generalization of the equations of Poisson.)

Remark 7.2. The non-existence of instantaneous actions (far-action) is true not only for electromagnetic fields (H. Hertz) but also for gravitation. Indeed, even if we accept that location of space bodies is given by the nature and *known* by astronomers, those bodies (planets and stars) are in motion, so the gravitational field is constantly changing. Due to the time uncertainty which may cause errors of up to 30 000 km for measurements by the rays of light if possible delays are in the range of 0.1 s in a computer or in transmission channels [6, p. 1568], the momentary distribution of space bodies, thus, the momentary gravitational field, cannot be precisely specified by a fixed set of formulae or equations.

Remark 7.3. Postulate 2, if assumed independent of $v \neq 0$, is questionable. Indeed, it follows from (31): $\varepsilon G(x) = \mu\xi_{\tau\tau} = \mu x_{tt}\,\beta^3$. For a still system (K), we have $v = 0, \beta = 1$, thus, $G(x) = (\mu/\varepsilon)x_{tt}$, so that the currently accepted postulate 2 presents a simple normalization condition $\mu/\varepsilon = 1$, yielding $G(x) = x_{tt}$ for a still system (K) with properly chosen and currently accepted units, in which the second law of Newton is valid. If the principle of relativity holds also for uniform accelerations, then intensity of gravitation $G(x)$ and the proper acceleration x_{tt} in a moving system (k) should not depend on v, so that $(\mu/\varepsilon)\beta^3 = 1$, $\mu = \varepsilon\beta^{-3} < \varepsilon$, if $v > 0$, thus $\beta > 1$. Hence, the inert mass μ and the gravitational mass ε (a "charge" with respect to a field) are *different* for $v > 0$. At $v = 0$, we have $\beta = 1$, thus $\mu = \varepsilon$ and $x_{tt} = \xi_{\tau\tau}$ with $\tau \equiv t$ according to (15). This effect we observe in everyday life as bodies with different weights (masses) falling in vacuum (and in the air if its resistance to the falling bodies is negligible) with the visibly same accelerations since $\beta \cong 1$ for $(v/V)^2 \cong 0$ in (15). On the contrary, if the inert and gravitational masses are assumed to be *exactly* equal and not depending on the velocity of the system in which they are considered, then the principle of relativity, Law 1, cannot hold even for uniform accelerations in rectilinear motions. Here we accept the principle of relativity, Law 1, as a valid postulate for such motions, until the contrary is proven by experiments. Thereby the inert μ and gravitational ε masses are equal only at rest and otherwise related as $\mu = \varepsilon\beta^{-3}$ which hopefully can be proven by experiments, as was done by H. Hertz for the non-instantaneous propagation of the electromagnetic field, and argued by Albert Einstein also for gravitational fields in [15].

8. The Lorentz transformations and wave invariants

First, we reproduce Einstein's rendition of the links with the Lorentz transformations in quotations from [16] translated from [2, tome II, pp. 416–423], in Einstein's notations with our remarks and formula numbers. In [16], Einstein writes: "The special theory of relativity came out of the Maxwell equations of electromagnetic field. It so happened that even in derivation of principal laws and notions of mechanics, a significant role was played by the laws of electromagnetic field. The question about independence of these laws is quite natural, since the Lorentz transformations, being, as a matter of fact, the basis of the special theory of relativity, are not, in themselves, linked directly to the theory of Maxwell and because we do not know to what degree the notion of energy in the theory of Maxwell may change under the influence of molecular physics. In considerations given below, we shall take as a basis, apart from the Lorentz transformations, only the laws of conservation of energy and impulse.

We start with an attempt to justify the expressions for the energy and the impulse of a material particle in a well known way. The fundamental invariant of the Lorentz transformations is

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2, \tag{32}$$

or

$$\mathrm{d}s = \mathrm{d}t(1-u^2)^{0.5}, \tag{33}$$

where

$$u^2 = (\mathrm{d}x/\mathrm{d}t)^2 + (\mathrm{d}y/\mathrm{d}t)^2 + (\mathrm{d}z/\mathrm{d}t)^2 = u_1^2 + u_2^2 + u_3^2. \tag{34}$$

If components of the contravariant vector $(\mathrm{d}t, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$ are divided by $\mathrm{d}s$, then we get the vector

$$(1-u^2)^{-1/2}, u_1(1-u^2)^{-1/2}, u_2(1-u^2)^{-1/2}, u_3(1-u^2)^{-1/2}. \tag{35}$$

Let the vector $(\mathrm{d}t, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$ be directed along the world line of a particle with mass m. We shall get the vector related with its motion, if we multiply by m the 4-vector of velocity which is just written above. Thus, we obtain

$$(\eta^\sigma) = [m(1-u^2)^{-1/2}, mu_i(1-u^2)^{-1/2}], \tag{36}$$

where index i takes the values from 1 to 3. Ignoring the third power of velocity, we can express the components of this vector as follows

$$(\eta^\sigma) = [m + 0.5\, mu^2, mu_i]. \tag{37}$$

Space components (η^σ) in this approximation coincide with components of the impulse in classical mechanics, and the time component, up to an additive constant m, coincides with the kinetic energy of a material point.

Returning again to the exact expression for (η^σ), it is natural to consider

$$mu_i(1-u^2)^{-1/2}, \tag{38}$$

as impulse, and

$$m[(1-u^2)^{-1/2} - 1] \tag{39}$$

as the kinetic energy of the particle. But how do we have to interpret the time component $m(1-u^2)^{-1/2}$, the expression of which has quite a real sense? Here, it is reasonable to directly ascribe to it the sense of energy, and thus, to ascribe to a still particle the *energy of rest m* (mc^2 in usual units).

This conclusion, of course, cannot be considered as a proof, since it does not follow at all that under interaction of several identical particles with each other this impulse agrees with the law of conservation of impulse, and this energy—with the law of conservation of energy; a priori, it could happen that other expressions for velocity enter the laws of conservation.

Besides, it is not quite clear what is to be understood under the *energy of rest*, since the energy is defined only up to an uncertain additive constant; in this respect, however, it is worth noting the following. Any system can be considered as a material point, until we do not deal with any other processes, apart from changes of translational velocity as a whole. However, there is quite clear sense in consideration of changes of the energy at rest in the case of processes that cannot be reduced to a simple change of translational velocity. Then the interpretation given above requires that in such processes the mass of a material point be changing as energy at rest; this requirement, of course, needs a proof".

Remark 8.1. First of all, the word "impulse" is used above in the sense of "momentum" as in [13], and exact expression in (39) equals $0.5mu^2$ of (37), up to the third order of small u, which presents the kinetic energy of motion if m in (37) is considered as *energy of rest*. Now, the terms in (32) are to be understood as squares of small segments (differentials): $\mathrm{d}x^2 = (\mathrm{d}x)^2 \neq d(x^2) = 2x\mathrm{d}x$. In this sense, the terms in (34) are projections of a 3D vector of velocity u obtained by division of (32) by $\mathrm{d}t^2$, and then taking the square root of (32) presented in (33). Further, there is a *rule of dimension* (denomination) in physics (not to be confused with geometric or topological dimension), which states that in any formula related to *physical* values, denominations (units) of all additive terms must be the same. For this reason, the term $\mathrm{d}t^2$ in (32) should include the factor $V^2 \neq u^2$ (the speed of a signal propagating uniformly in all directions: a spherical wave carrying the time component $\mathrm{d}t^2$; see (50)–(52)). Indeed, in the CGS system the term $\mathrm{d}t^2$ has denomination s^2 whereas other terms in (32) have denomination cm^2. The terms u^2, u_i^2 $(i = 1, 2, 3)$ in (34) all have denomination of velocity $(\mathrm{cm/s})^2$, thus, in relations (32)–(34) the *rule* will be respected, if we assume in (32) the factor $(V\,\mathrm{cm/s})^2 = 1$ that multiplies $\mathrm{d}t^2$. In fact, Albert Einstein tells about it noting "the *energy of rest m* (mc^2 in usual units)"; see above. However, if $V^2 = c^2 = 9 \times 10^{20}$ $(\mathrm{cm/s})^2$, then setting it equal to 1 distorts the scale of dimensions (comparative units), although such a normalization is used in the literature; see, e.g. [9, Ch. IX].

Remark 8.2. Taking $c^2 = 1$ is equivalent to dividing (32) by $\cong 10^{21}$ which justifies the approximation up to the third order of velocity u in the expansion of $(1-u^2)^{-1/2}$, used by Einstein in (37), since after such a division velocities u_i are very small, yielding precision of the order of $\cong 10^{-63}$. However, it is done at a cost of essential distortion in spatial and temporal scales relevant to the terms in (32).

Remark 8.3. The Lorentz invariant (32) describes the conservation of the length (linear element) in 4D geometry, being, thus, purely *geometric*. If we discard $\mathrm{d}t$ and write plus (+) instead of minus (−) before the squared segments on the right in (32), we get the usual invariant in the Euclidean 3D geometry (the diagonal of a parallelepiped), independent of time. In (35), Albert Einstein considers the components of velocity corresponding to small segments of a linear element of the 4D geometry. Then, in (36) those components are multiplied by a mass m, presenting the "components of an impulse in classical mechanics" (momentum), thus "the kinetic energy of a material point" which coincides "up to an additive constant m" with "the time component", completing the passage from the *geometric* Lorentz components in (32) to *dynamic* Einstein's components in (35)–(38) with the kinetic energy in (39). These new dynamic components of momentum and of kinetic energy are used by Einstein to derive from the Lorentz invariant (32) the conservation laws for impacts of material points and the mass–energy relation $E_0 = m$.

Further, Einstein writes: "Now we shall demonstrate that if the laws of conservation of energy and impulse are valid in all coordinate systems linked with each other by the Lorentz transformations, then the energy and impulse are really defined by the above mentioned formulae, and supposed equivalence of the mass and energy at rest also exists.

Let us start from the simple kinematical consequences of the Lorentz transformations:

$$t = (t' + vx')(1 - v^2)^{-1/2}, \qquad x = (x' + vt')(1 - v^2)^{-1/2}, \qquad y = y', z = z', \tag{40}$$

where v is the relative velocity of coordinate systems K and K'. The same relations hold also for differentials dx, etc. Making the corresponding calculations, it is easy to get the law for transformation of the components of velocity:

$$u_1 = (u_1' + v)/(1 + u_1'v), \qquad u_2 = u_2'(1 - v^2)^{1/2}/(1 + u_1'v), \qquad u_3 = u_3'(1 - v^2)^{1/2}/(1 + u_1'v). \tag{41}$$

From this, it follows ...” [expressions are given for the values in (33), (34) through the velocities u_i' ($i = 1, 2, 3$) of (41)]. Further, Einstein considers a couple of particles of equal mass with velocities in K' equal and opposite in direction and derives expressions for respective velocities of those particles [16], or [2, tome II, p. 419]. Then Einstein writes:

“Let us pass now to the essence of the problem. Suppose that the impulse and energy of a material point are given by expressions of the form

$$I_\nu = mu_\nu F(u), E = E_0 + mG(u) \quad (\nu = 1, 2, 3), \tag{42}$$

where F and G are universal even functions of velocity u, vanishing as $u = 0$. Then $mG(u)$ will represent the kinetic energy, E_0 the *energy at rest* of a material point, and m the *mass at rest*, or simply the mass. Here it is assumed that impulse and energy of a *point-wise mass* do not depend on the direction of motion and on the orientation of the *point-wise mass* with respect to its velocity. Further, it is assumed that the expressions for impulse and energy contain *one and the same* constant mass m. Later, we shall find a partial justification thereof.

Now, let us consider the elastic non-central impact of two particles of equal masses. One can always choose a system of coordinates K' so that with respect to that system the velocities of masses before the impact would be equal to each other in value and opposite in direction. What are the velocities of particles after the impact with respect to the system K'? If the velocities after the impact would not be, as before, equal and opposite in direction, then it would contradict the law of conservation of impulse. If the equal in value velocities of both masses after the impact would not be equal to the respective velocities before the impact, then it would contradict the law of conservation of energy. These conclusions do not depend at all on a particular form of dependence of the impulse and energy on the velocity. Thus, the impact changes only direction of motion of two point-wise masses with respect to system K'. For short, this can be expressed as follows: a couple of particles before an impact is transformed after the impact into a couple of particles with the same velocity u'”. Equations for velocities before and after an impact follow, and Einstein writes: “These equations are true for the general case of elastic impacts of equal masses, and have the form of conservation laws; therefore, it can be considered proven that there are no other symmetric or anti-symmetric functions of components of velocity which in the considered case of elastic impact of two identical point-wise masses would produce similar relations. Accordingly to this, we have to consider $mu_i(1 - u^2)^{-1/2}$ of (38) as impulses and $m[(1 - u^2)^{-1/2} - 1]$ of (39) as kinetic energy of a particle.

Let us proceed now with the proof that the mass equals the *energy of rest*. For the full energy E of a moving particle, we have to take the expression

$$E = E_0 + m[(1 - u^2)^{-1/2} - 1], \tag{43}$$

whereby we shall assume that E_0 (energy of rest) and m may be changing in the case where the interaction of point-wise masses is not *elastic*.

Now let us consider the non-elastic impact of two particles with equal masses and energies at rest, which before the impact formed a *couple of particles* with respect to system K' with equal in value and opposite velocities. Further, we shall assume for simplicity that the particles at impact undergo the same internal changes. From the law of conservation of impulse, it follows that in system K' the final velocities of particles must be equal in value and opposite in direction. The law of conservation of energy in systems K' and K implies that the full energy defined by (43) must be equal in systems K' and K for a couple of particles: $E' = E$. Einstein justifies it by consideration of corresponding equations for velocities and energy [16], or [2, tome II, p. 421]. Further, “for the couple of particles before and after the impact, those equations can be rewritten in the form:

$$E_0 - m + m(1 - u^2)^{-1/2}(1 - v^2)^{-1/2} = E_0' - m' + m'(1 - u'^2)^{-1/2}(1 - v^2)^{-1/2}, \tag{44}$$

$$E_0 - m + m(1 - u^2)^{-1/2} = E_0' - m' + m'(1 - u'^2)^{-1/2}. \tag{45}$$

Multiplying the last equation by $(1 - v^2)^{-1/2}$ and subtracting the result from (44), we obtain

$$[(E_0' - E_0) - (m' - m)][(1 - v^2)^{-1/2} - 1] = 0, \tag{46}$$

or

$$E_0' - E_0 = m' - m. \tag{47}$$

Thus, the energy of rest after non-elastic impact changes additively, like the mass. As to the energy of rest, it is defined, as follows from the very notion of energy, only up to an additive constant, and we can impose the condition that E_0 become zero together with m:

$$E_0 = m, \tag{48}$$

which is a proof of the principle of equivalence of the inert mass and the energy of rest.

From the law of conservation of the x-component of the impulse, it follows (for a non-elastic impact): $m(1-u^2)^{-1/2} = m'(1-u'^2)^{-1/2}$. This relation follows also from Eqs. (45), (47) obtained from the law of conservation of energy. If from the very beginning, we had assumed that in the expression of impulse a constant mass enters, then using similar considerations, it could be demonstrated that after non-elastic impact the "impulse mass" changes in the same way as "energy mass". This represents a partial justification of the assumed equality of both masses.

Our results can be summarized as follows. If after an impact of point-wise masses the laws of conservation are satisfied in all (Lorentzian) systems of coordinates, then from this alone it follows the known expressions for impulse and energy, as well as the validity of the principle of equivalence of the mass and energy of rest.

Professor Birkhoff has brought my attention that in his book "Relativity and Modern Physics", written jointly with professor Landger, similar considerations are presented about impacts of particles, and also about energy and impulse. This notwithstanding, it seems to me that the derivation given above represents certain interest.

In particular, in the just mentioned book the notion of *force* is essentially employed, which in relativistic theory does not have such a clear sense as in classical mechanics. This is due to the fact that in the latter the force must be considered as a given function of coordinates of all particles, which obviously is impossible in the relativistic theory.

Besides, I avoided making any assumptions about transformational properties of the energy and impulse with respect to the transformations of Lorentz".

Discussion. Comparing (40) with Einstein's transformations (15), one can see that (40) are the inverse of transformations (15) if we set $V = 1$, and denote $(1-v^2)^{-1/2}$ as β, and (t', x', y', z') as (τ, ξ, η, ζ). Thus, system K' is, in fact, identical to (k) from [1]; see Sections 2 and 3 above. Also, components of velocity in (41) coincide with the formulae given in the "Theorem of addition of velocities" in [1, Section 5] for a point moving in (k) with constant velocity $w = (w_\xi, w_\eta, w_\zeta) = (u_1, u_2, u_3)$ of K'. This means that the Lorentz transformations represent a particular case of Einstein's transformations (15) which were derived independently using the time synchronization conditions (1)–(2); see Sections 2 and 3 above. For this reason, it is interesting to formulate the Lorentz invariant (32) in terms of Einstein's transformations (15) which respect the rule of dimension and would clarify the physical sense of the Lorentz invariant. Squaring the time–space coordinates in (15) and introducing the *measured* velocity $p = \mathrm{d}\xi/\mathrm{d}t = -\beta v = \text{const}$, cf. [6, Lemma 9.1], instead of postulated (usually unknown) relative velocity $v = \text{const}$, whereby

$$v/V = -(p/V)[1+(p/V)^2]^{-0.5}, \qquad \beta(v) = [1-(v/V)^2]^{-0.5} = [1+(p/V)^2]^{0.5} = \gamma(p) \geq 1, \tag{49}$$

we get the expressions for spherical wave propagation in Einstein's β-representation (15), and in its γ-representation [6, p. 1567], based on *measured* velocity p, as follows:

$$0 = \xi^2 + \eta^2 + \zeta^2 - V^2\tau^2 = \beta^2(x-vt)^2 + y^2 + z^2 - V^2\beta^2(t - vx/V^2)^2 \tag{50}$$

$$= (\gamma x + pt)^2 + y^2 + z^2 - V^2(\gamma t + px/V^2)^2 = x^2 + y^2 + z^2 - V^2t^2 = 0. \tag{51}$$

This means that the observed in (k) spherical waves (50) are identical to initial spherical waves in (K), the last equality (51), thus, Einstein's transformations preserve the identity of initial and observed light propagation waves—a version of synchronization condition equivalent to (1)–(2). Opening the parentheses in (50)–(51), the reader can verify that parameters β, v, γ, p algebraically cancel out, thus, (50)–(51) are valid also for *variable* $v(t), p(t)$, and the *observed*, (50) left, and *initial*, (51) right, waves depend only on the signal propagation velocity V (the speed of light in Einstein's consideration), in agreement with the physical sense of observation process and the synchronization arrangement in Sections 2 and 3 above.

The simple form of the wave equations in (50)–(51) is due to the choice of zero initial data; see [1] or [6, Sec. 7.1]. For arbitrary initial conditions, zeros in (50)–(51) should be replaced by a constant, and squares of coordinates by differences $(x-x_0)^2, \ldots, (t-t_0)^2$, which in differential form with changed sign in (51) yield the Lorentz invariant (32) for $V = 1$, cf. [9, Ch. IX, (9.3.7)–(9.3.9)]. In addition, if we take the values $\xi = \eta = \zeta = 0$ in (50) at some moment $\tau = t' > 0$, then we would have $V^2t'^2 = V^2t^2 - x^2 - y^2 - z^2 \neq 0$, which in differential form can be written as follows

$$\mathrm{d}s'^2 = V^2\mathrm{d}t'^2 = V^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2, \quad (p, v, V = \text{const}). \tag{52}$$

Now, if $t' = 0$, as in Einstein's setting $t = 0$ for $\tau = 0$ at the origin, then $\mathrm{d}s'^2 = 0$, and we return from invariant of (52), similar to (32), to the wave equations in (50), (51) for small values $\mathrm{d}x, \mathrm{d}y, \mathrm{d}z, \mathrm{d}t, \mathrm{d}\xi, \mathrm{d}\eta, \mathrm{d}\zeta, \mathrm{d}\tau$ counted from some common zero point $x = y = z = \xi = \eta = \zeta = 0$ at $t = \tau = 0$. From (50)–(51), it follows that propagation of waves is invariant also with respect to the measured velocity $p = \text{const}$, which is masked in formula (32). Invariant (52) directly relates to the wave propagation (50)–(51) for *arbitrary* information transmitting signal at constant velocity V, and expressly shows some contingencies that may be important in real life processes. However, geometric invariant (32) with normalization condition $V = 1$ (cm/s) applies to all observation signals, hence, it supports the concept of relativity affecting *all* interacting processes, linked by *any* signals, not just by rays of light—the point of view advanced in [6].

There is strong temptation to regard the metric invariant (32) as a pillar of the general relativistic 4D geometry that defines the structure of the universe. Albeit the importance of this invariant, magnified by the beauty of quaternionic considerations [9, Ch. IX], is quite clear, it is worth noting that this invariant relates only to signals propagating as spherical waves at a constant speed $V = 1$ through isotropic media. In nonlinear relativity at variable velocities, the Lorentz invariant and Einstein's transformations (15) should be modified. However, they are applicable to piece-wise linear approximation of trajectories through *measured average* velocities p_n over small intervals of time dt_n; see [4,5].

Comparing the principle of the mass-energy equivalence (48) with the relations (21), (22) and (29)–(31), it is clear that $E_0 = m = m_0 = \varepsilon = \mu$ at $v = 0$, that is, the "energy of rest" E_0 represents the scalar static mass μ in a still system (K). It presents itself in the second law of Newton at the *very start* of a motion from $v = 0$, and then it turns into $m_0 = \mu\beta^2$ or $m = \varepsilon = \mu\beta^3$ depending on direction and on signals through which the information about the motion is transmitted. Hence, the *observed* energy is *relativistic* as well as the fields of forces, electromagnetic, gravitational, or others. Also, the term "information transmittal" includes the *real physical actions* which are transmitted by specific signals propagating at their specific velocities $V < c$ (forces of impact, electric current, air flow, muscle contraction, blood circulation, etc.) that support the normal evolution of physical, chemical and life processes conditioned on those signals at $V > |v|$.

9. Relativistic identification of the gravitational field

In [15], see Section 7 (Part II) above, Albert Einstein argues against Newtonian far-action (instantaneous information transmittal) and absolute time. The same arguments can be called against the use of the Lorentz and Einstein transformations and invariants defined for *constant* relative and signal velocities for investigation of motions and processes in accelerated systems moving with *variable* velocities under gravitation. For this reason, it is expedient to concentrate on experimental measurements that can provide a realistic picture of the gravitational field with respect to the time t of observer, system (K).

If we consider the point x, y, z as a known point of observation in a still frame (K) and assume that the value of a constant velocity v is known and initial conditions satisfy the equations specified in (15), then Einstein's transformations (15) completely describe the time and coordinates of a point (ξ, η, ζ, τ) in the moving frame (k) *if observed in the still system* (K) [1]; or [2, tome I, pp. 15–18] as functions of (x, y, z, t, V, v). In reality, if that point $\xi(.) \in (k)$ represented a rocket, asteroid or spacecraft, then initial conditions of the motion may be unknown, and also velocity v is neither known nor constant. In such cases, accurate observation of that body $\xi(.)$ is possible only after the velocity v and actual position at some moment in time are identified assuming that the speed V of the signal (carrier of information) is known and constant, as specified by the principle of the constancy of the speed for rays of light in Law 2, Section 2. In the general case of variable velocity $v(t) \neq$ const, Einstein's transformations (15) can be used if *average* velocities are introduced on a discretized trajectory, which velocities are identified over the pieces where the observation of the moving body need to be supported.

9.1. Design of experiments

Consider a still point x_0 on the X-axis of a still frame (K) at which point a source of light is fixed beaming along the X-axis with short pulses of light. The reader can imagine the origin of (K) at the center of Earth, the point x_0 at the top of a hill at a place with clear air and good weather, the axis $0x$ pointing to the outer space where an asteroid $\xi(t, x_0)$ is observed at x_0 moving along the right line $0x$ with $y = z = 0$. Short pulses can be extracted from continuous beams of light with a thin evenly perforated disc with windows (openings, gaps) of 1 mm wide and closures of the same or different widths rotating with a high speed in a vacuum enclosure. To control the pulses, the vertical shaft of the disc can be turned at small angles to the vertical and the speed of rotation can be varied. The stand is similar to the setup of Fizeau [17] and Cornu [18]; see also [11, pp. 1276–1277] for details and calculations.

Consider the time moments $t_n = n\Delta t$, $n = 0, 1, \ldots$ at which pulses are sent to the asteroid and the later moments $t'_n = t_n + \Delta t_n$, at which reflected light of those pulses is received at the same point x_0 where the source of light is located. Here, the increments Δt and Δt_n are small finite time differences such that the ray of light (pulse) sent at t_n is reflected and received back at the moment t'_n, $n = 0, 1, \ldots$. The length of discretization interval Δt can be varied at will through disc control [18].

9.2. Computation of the average velocities of (k) as observed in (K)

We shall use the scheme of Einstein, with a difference that, instead of sending a ray $\xi \to x' \to \xi$ (there are no people on an asteroid who could send a ray to the point x'), in order to synchronize the timing of events at $\xi \in (k)$, on the asteroid, and at $x' \in (K)$, see Eqs. (7), (13), (14), (18), the rays are sent in *opposite directions* $x_0 \to \xi \to x_0$, to measure the actual distances to the points of reflection of the rays from the moving asteroid, whatever its velocity $w(t)$ may be. We assume that $w(t) > 0$ corresponds to the direction of increasing x, so that the asteroid moves away from the Earth.

At a moment t_n when a pulse is sent, the body (asteroid) is at some unknown distance from x_0. When the pulse is reflected, the body is at a greater distance x_n which can be computed, upon reception of reflected ray, by the formula $x_n = 0.5V\Delta t_n$, although at the moment $t'_n = t_n + \Delta t_n$ of reception, the body will be at still greater (unknown) distance from x_0. Sending the next pulse at the moment t_{n+1}, we can compute in the same way $x_{n+1} = 0.5V\Delta t_{n+1}$, yielding $\Delta x_n = x_{n+1} - x_n = 0.5V(\Delta t_{n+1} - \Delta t_n)$ where time increments are measured at x_0. The last equation holds for all $n = 0, 1, \ldots$

and any constant speed V of the pulse signal. Between two subsequent reflections, the body has moved a distance

$$\Delta x_n = x_{n+1} - x_n = 0.5V(\Delta t_{n+1} - \Delta t_n) = \int_a^b w(t)\mathrm{d}t = w_n(b-a) \tag{53}$$

$$= w_n(t_{n+1} + 0.5\Delta t_{n+1} - t_n - 0.5\Delta t_n) = w_n(\Delta t + 0.5\Delta t_{n+1} - 0.5\Delta t_n). \tag{54}$$

Here $w(t)$ is the unknown velocity of the body with respect to the time t *as observed from the still frame* (K) on Earth, and in (53) we have used the first mean value theorem for integrals with w_n as notation for yet unknown average velocity on the interval (a, b) specified in (28). Comparing the entries in (53), (54) where $x_n = 0.5V\Delta t_n$ $(n = 0, 1, \ldots)$ are already computed, we find

$$w_n = V(\Delta t_{n+1} - \Delta t_n)/(2\Delta t + \Delta t_{n+1} - \Delta t_n) = 2(x_{n+1} - x_n)/(2\Delta t + \Delta t_{n+1} - \Delta t_n), \tag{55}$$

which allows us to compute w_n through measurements of the time increments in (55). We have $\Delta t_{n+1} > \Delta t_n$ since $x_{n+1} > x_n$, so that

$$2\Delta t + \Delta t_{n+1} - \Delta t_n = 2\Delta t + \varepsilon, \quad \varepsilon > 0, \tag{56}$$

and if $\Delta t \to 0$, then $\varepsilon = \Delta t_{n+1} - \Delta t_n \to 0$, since the whole sequence of pulses contracts into one single pulse. In this case, from (29) it follows that $w_n = \Delta x_n/(\Delta t + 0.5\varepsilon)$, yielding

$$\Delta x_n/\Delta t = w_n\,(\Delta t + 0.5\varepsilon)/\Delta t > w_n, \quad n = 0, 1, \ldots, \tag{57}$$

and as $\Delta t \to 0$ we get, in the limit: $\mathrm{d}x/\mathrm{d}t = w(t)[1 + 0.5\lim(\varepsilon/\Delta t)] = w(t)$, since $\varepsilon/\Delta t$ is positive, so its limit must be zero according to the definition of the mean value $w_n > 0$ in (53). If $\mathrm{d}x/\mathrm{d}t = w(t) = p = \text{const}$, then $w_n = p$, and we return to the model of Einstein with $v = \text{const}$, for which transformations (15) hold. It implies that a mapping exists between the constant parameters p and $v \neq p$, cf. wave invariants (50) and (51), Section 7.

9.3. The γ-representation

Equating p and time derivative of ξ in (15), we have

$$\mathrm{d}\xi/\mathrm{d}t = -\beta v = -v[1 - (v/V)^2]^{-0.5} = p, \quad \text{if } v = \text{const},\ p = \text{const}. \tag{58}$$

Solving (58) for v, we get

$$v = -p[1 + (p/V)^2]^{-0.5} = -p\gamma^{-1}(p), \qquad \beta(v) = \gamma(p) = [1 + (p/V)^2]^{0.5}, \tag{59}$$

which yields, after the substitution of $v(p)$, $\beta(v)$ into (15)

$$\tau = \beta(t - vx_0/V^2) \equiv \gamma(p)t + px_0/V^2, \quad \gamma(p) = [1 + (p/V)^2]^{0.5}, \tag{60}$$

$$\xi = \beta(x_0 - vt) \equiv \gamma(p)x_0 + pt, \quad x_0 = \text{const},\ p = \mathrm{d}x/\mathrm{d}t = \mathrm{d}\xi/\mathrm{d}t = \text{const}. \tag{61}$$

It follows from (58) that $v = 0$ if $p = 0$, and if $p \neq 0$, then $v^2 < p^2$ and $v^2 < V^2$, thus the physical condition $|v| < V$ assumed in [1, Section 4], cf. Section 4, is automatically satisfied. The identities in (60)–(61) at the right-hand side provide the γ-representation for motions with constant velocities which is based on directly measured derivative in (61). If we consider discretization of motion with varying average velocities w_n between adjacent pulses, it is clear that over each interval $(a, b) = (t_n + 0.5\Delta t_n, t_{n+1} + 0.5\Delta t_{n+1})$ in (53)–(54) the motion with variable speed $w(t)$ is represented by the uniform motion with constant average velocity w_n, and relativistic transformations (60)–(61) with constant parameters $v_n, p_n = w_n$ $(n = 0, 1, \ldots)$ of (55) are valid over those intervals. Computed by (55) values of w_n can be substituted for p into (58) to compute v_n, β_n whereupon transformations (15) can be used. However, it is much simpler to use $p_n = w_n = \Delta x_n/(b-a) > 0$ in (53)–(54), then compute $\gamma_n = \gamma(p_n)$ from (60), and estimate the trajectory making use of the expressions in (60)–(61) at right for the γ-representation. We see that relativistic transformations (15), (60)–(61) derived for the relative velocity $v = \text{const}$ can be used with discretization and on-line observation of the actual motion in appropriate segments along its trajectory.

9.4. Computation of the average accelerations in (k) as observed in (K)

Once $p_n = w_n$ $(n = 0, 1, \ldots)$ of (55) are computed, we can determine approximate accelerations $a_n \cong \Delta w_n/\Delta t$ which define the actual gravitational field, if there are no other fields or forces acting on a material point in the accelerated system (k). If there are other forces, then a_n defines the density of a combined action (force) on the material point that passes through x_n at the moment t_n. The current value of a_n can be computed as follows. According to (53)–(55), the value w_n is computed at the moment $t'_{n+1} = t_{n+1} + \Delta t_{n+1} + \delta = (n+1)\Delta t + \Delta t_{n+1} + \delta$, where δ is the unknown delay in measurement and computation. Similarly, the next value w_{n+1} is computed at the moment $t'_{n+2} = t_{n+2} + \Delta t_{n+2} + \delta = (n+2)\Delta t + \Delta t_{n+2} + \delta$, yielding the average acceleration

$$a_n = (w_{n+1} - w_n)/(t'_{n+2} - t'_{n+1}) = (w_{n+1} - w_n)/(\Delta t + \Delta t_{n+2} - \Delta t_{n+1}), \tag{62}$$

where uncertain delays are canceled out because the measurements and computations of successive w_n at the moments t'_{n+1} are made with the same instruments. If measurements are made by radar at $V = 300\,000$ km/s, then $\Delta t_n = 2x_n/V$ are small and we get a simple relation $a_n \cong (w_{n+1} - w_n)/\Delta t$ for a fixed time difference Δt between pulses. We see that a field of forces can actually be measured if a material point $\xi \in (k)$ moving with (k) can be observed from (K).

Using the gravitational mass $\varepsilon = \mu\beta^3$ from (31) that follows from Einstein's energy relation (29), we obtain the force of gravity (weight) as $P_n = \varepsilon G_n(x) = \varepsilon a_n = \mu\beta^3 a_n$. If $v \to V$, then $\beta \to \infty$ bringing the illusory black hole effect $P_n \to \infty$, $\forall n$, at finite values of intensity of gravitation $G_n(x)$. However, if the signal velocity $V^2 \le w^2 = p^2 = \beta^2 v^2$, in case $V^2 - v^2 \le v^2$, see (58), that is $v \ge 2^{-0.5}V \cong 0.7071V$, then the signal (rays of light or radar) cannot catch up with the asteroid ξ moving at velocity v. In this case, the observation of w_n by reflected rays of light or radar is impossible, which puts a limit for observable intensities of gravitational field $G_n(x)$ by the proposed method. At smaller velocities $v < 2^{-0.5}V \cong 0.7071V$, the variable intensity $G_n(x)$ of the gravitational field is given by the measured accelerations a_n according to (62).

Remark 9.1. The method allows us to experimentally determine the combined intensity of all fields (gravitational, electromagnetic, the pressure of light, or other media) which are accelerating and/or decelerating the motion of a material point. It does not allow us to distinguish a portion due to specific action of some particular field unless it is known a priori that all other fields are not present or, if present, supply only marginal negligible effects in the chosen direction $0x \in (K)$ of observation. According to (55), the measured intensity $G_n(x)$ does *not* depend on the point x_0 of observation, thus, presenting the actual gravitational field *everywhere* along the axis $0x$ of the experimental measurements. Moreover, relations (55), (62) implicitly account for the fact that information transmittal takes not only the time but also some energy in the process of signal propagation.

10. Local invariants of relativistic dynamics and gravitation

All motions and processes in nature and technology are evolving under close interaction of different forces, fields and components assured by the proper transmission of information from one component or process to another. This transmission takes time and is realized through certain transmission signals propagating at different velocities.

For this reason, we must introduce the information transmittal signals into the Lorentz invariant (32) of relativistic 4D geometry. Let us write this invariant in the form:

$$\mathrm{d}s^2 = V^2\mathrm{d}t^2 - \mathrm{d}x^2 - \mathrm{d}y^2 - \mathrm{d}z^2\,. \tag{63}$$

Here V is the speed of a concrete information transmittal signal with which the points x, y, z are observed and/or interacting within a motion or a process evolving in time. The relation (63) complies with the physical rule of dimension. If $V = 1$, we return to the original Lorentz invariant (32). If $\mathrm{d}s = 0$, we obtain Einstein's wave equations (50)–(51) in their β- and γ-representations for the proper x, y, z, t and relativistic ξ, η, ζ, τ space–time coordinates with the synchronized time τ. Since parameters β, v, γ, p algebraically cancel out in (50)–(51), relations (50)–(51) and invariant (63) are valid in the proper x, y, z, t and relativistic (observed) coordinates ξ, η, ζ, τ, for *variable* velocities $v(t), p(t)$ in the *observed*, (50) left, and *initial*, (51) right, waves in (50)–(51). Hence, relation (63) with $V = \text{const}$ is valid in *any* synchronized space–time coordinates.

Since V is the speed of an information transmittal signal, relation (63) has transparent physical sense of an *imprecision* $\mathrm{d}s^2 > 0$ due to time delay in observation of the Euclidean length of a vector $(\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$. Indeed, if $\mathrm{d}s^2 = 0$, then its length is exactly measured with a signal propagating at a known speed V by the time $\mathrm{d}t$ of its propagation along this length. If $\mathrm{d}s^2 = \text{const} > 0$, it means a constant universal imprecision for *all* vectors measured by the signals of the same speed V. If $V = c$, the speed of light, it means that the rays of light are used for the measurements as in the model of Einstein; see Sections 2 and 3. This value $V = c$ is used in some sources, see, e.g., in [9, Ch. IX, Sections 4–5]; or [10, pp. 636–640]. However, the fixing of V, it being $V = 1$ (Lorentz) or $V = c$ (other sources), albeit correct for mathematics, effectively excludes from relativistic considerations almost all natural and physical processes. It completely excludes processes in biology, medicine, chemistry, economics, aviation, water and climate phenomena, and many processes in engineering, communications and computer science. Life, technology and economy all depend on the precise and timely transmittal of information, *not* to observe something but to *allow* a system to function and its processes to evolve. Information is transmitted by signals provided by specific processes, *not* necessarily rays of light or radar. Moreover, the rays of light or radar do not propagate and are not carriers of information in living organisms, chemical solutions, in engines and transport vehicles, in earthquakes and tsunamis, etc. For this reason, we consider in this paper realistic signals propagating at their proper velocities. For such cases, it is inappropriate to use one or several fixed relativistic invariants, so we consider *sets* of invariants corresponding to different values of V for the relativities actually existing in nature, life and technology.

If $\mathrm{d}s^2 \ne \text{const}$, it means that the measuring or computation system is *unstable* and yields different and imprecise results for fixed lengths, which prevents rigorous theoretical or experimental studies. However, if this non-constancy is bounded within small intervals of imprecision, then suitable theories can be developed to reflect realistic situations.

Remark 10.1. It is important to understand that invariant (63) is *local*, as well as all considerations below, and *non-crisp* but rather presents a *continuum of soft intervals* [19] corresponding to time uncertainty in measurement and computation, cf. [20, pp. 2485–2488]. It is valid only over small increment $\mathrm{d}t$ of time for infinitesimally small length of a vector $(\mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$, but this is for *any* space–time point in the proper x, y, z, t and relativistic (observed) coordinates ξ, η, ζ, τ. This complies with the consideration of discretized trajectories in Section 9, leading to correct local results. It does not allow straightforward integration unless Einstein's relativistic transformations (15) are upgraded for variable velocities; see [4,5,20].

Introducing the same velocity vector u as in (34), we get, instead of (33), the equation

$$\mathrm{d}s = \mathrm{d}t(V^2 - u^2)^{0.5} = V\mathrm{d}t[1 - (u/V)^2]^{0.5} = \beta^{-1}V\mathrm{d}t, \quad |u| < V, \tag{64}$$

where

$$u^2 = (\mathrm{d}x/\mathrm{d}t)^2 + (\mathrm{d}y/\mathrm{d}t)^2 + (\mathrm{d}z/\mathrm{d}t)^2 = u_1^2 + u_2^2 + u_3^2. \tag{65}$$

If the point x, y, z in (63) is moving, then (65), identical to (34), presents the square of the length of its velocity vector u, and if we choose the axis $0x \in (K)$ along this vector u, we return to the model of Einstein, see Section 3, where the point x, y, z corresponds to the observed point $\xi \in (k)$ moving with the system (k) along the line of the vector u at the velocity $v \equiv |u|$. For this reason, in (64) we have $u = v$, the same v as in (15), thus, the square root of the bracket in (64) equals β^{-1} with the same β as in (15). Now the motion of the point x, y, z in (63)–(65) can be considered as a translational motion in accordance with the scheme of Einstein in Section 3 which represents this motion over any infinitesimally small interval of time with some imprecision $\mathrm{d}s^2 > 0$ as in (63).

If components of the vector $(\mathrm{d}t, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$ in (63) are divided by $\mathrm{d}s = \beta^{-1}V\mathrm{d}t$ of (64), then, instead of (35) we get the vector

$$\beta V^{-1}, u_1\beta V^{-1}, u_2\beta V^{-1}, u_3\beta V^{-1}. \tag{66}$$

The first component of this vector has dimension s/cm whereas the three last components are unit free. Operations with such vectors are possible if and only if the intermediate and final results have physical sense. After (35), Einstein wrote: "Let the vector $(\mathrm{d}t, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z)$ be directed along the world line of a particle with mass m. We shall get the vector related with its motion, if we multiply by m the 4-vector of velocity which is just written above". Using (66) instead of (35), this yields, instead of (36):

$$S = [m\beta^{-1}, mu_i\beta V^{-1}], \quad i = 1, 2, 3. \tag{67}$$

If $V = 1$ as in (32), the 4-vector S of (67) coincides with (36), whereby $mu_i\beta$ coincide with (38) as impulses, and $m\beta$ coincides with (39) as "the kinetic energy of the particle" up to an additive constant m which is deducted in (39). The vector S is *not* a free vector. As Einstein indicated, vector S should be directed along the world line, which simply means that, in our case, it should satisfy invariant (63). With the components of (66) or (67), substituted for $\mathrm{d}t, \mathrm{d}x, \mathrm{d}y, \mathrm{d}z$, relation (63) takes the form:

$$1 = V^2\mathrm{d}t^2/\mathrm{d}s^2 - \mathrm{d}x^2/\mathrm{d}s^2 - \mathrm{d}y^2/\mathrm{d}s^2 - \mathrm{d}z^2/\mathrm{d}s^2 = \beta^2 - \beta^2 V^{-2}\Sigma u_i^2, \tag{68}$$

thus, the vector S of (67) satisfies invariant (63) for any $m \neq 0$ because

$$\beta^2 V^{-2}\Sigma u_i^2 = \beta^2 - 1, \tag{69}$$

which is the identity since $\Sigma u_i^2 = \beta^{-2}V^2(\beta^2 - 1) = u^2$ for $\beta = [1 - (u/V)^2]^{-0.5}$ of (15). If $v \equiv |u| \to V$, then $\beta^{-1} \to 0, \mathrm{d}s \to 0$ in (64), and invariant (63) turns into Einstein's wave equation (51) on the right; the observation in this case cannot be achieved in finite time since $\tau \to \infty$ as $\beta \to \infty$; see (15) or Section 9.4 above. Using (59), the relation (69) can be written in the form:

$$u^2 = \Sigma u_i^2 = V^2(1 - \beta^{-2}) = V^2(1 - \gamma^{-2}) = p^2\gamma^{-2} < p^2, \tag{70}$$

which is equivalent to invariant (63) in terms of velocities, and shows that in motions along the world lines, i.e. satisfying invariants (63) or (69), the absolute values of *actual* velocities are *less* than absolute values of *measured* velocities: $|u_n| < |p_n| = |w_n|, \ \forall n$, since $u^2 = p^2\gamma^{-2} < p^2, |u| = |p/\gamma| < |p|$ as $\gamma > 1$ for $|p| > 0$. Since $|p_n| = |w_n|$ are computed by (55) and also $u \equiv v = -p\gamma^{-1}(p)$ by (59), $u_n = -w_n\gamma^{-1}(w_n)$ are known, $\forall n$.

Now we shall consider the motion along the right line tangent to the trajectory of a material point of mass m defined by the variable vector $u(t)$ within a small segment corresponding to the increment $\mathrm{d}t$ of the proper time of an observer located at a point x_0 on Earth as considered in Section 9 according to the model of Einstein in Section 3. It is convenient to use the γ-representation of Section 9.3 for the observed coordinates ξ, τ of (k) referred to the time t of a still observer in (K). With t_n, V, x_0 known and all Δt uniformly tending to zero, invariants (69) and (70) can be differentiated. The values $p(t)$ and $a(t) = \mathrm{d}p/\mathrm{d}t$ of (62) are measured for every t_n, and the values τ, ξ are calculated in (60)–(61). Thus, the observed process $(\xi, \tau) \in (k) : \xi(t, x_0), \tau(t, x_0)$, and all the terms in (58)–(69) are known. Resolving (60)–(61) for t, x_0, we get

$t = \gamma\tau - \xi p/V^2$, $x_0 = \gamma\xi - p\tau = \gamma\xi - p(\gamma t + px_0/V^2)$, yielding $x_0 = \gamma^{-1}(p)(\xi - pt)$ which can be used for identification of the observation point x_0.

Invariants (69) and (70) can be expressed in terms of accelerations. Since, in the limit of (62) as $\Delta t \to 0$, we have $a(t) = \mathrm{d}p/\mathrm{d}t$ and $\mathrm{d}\gamma/\mathrm{d}t = V^{-2}\gamma^{-1}pa(t)$, we get from (70):

$$p\mathrm{d}p/\mathrm{d}t = u\gamma^4(p)\mathrm{d}u/\mathrm{d}t, \tag{71}$$

and since $p = -\beta v = -\gamma u$, see (58)–(59), $a(t) = \mathrm{d}p/\mathrm{d}t = -\gamma^3\mathrm{d}u/\mathrm{d}t = -\beta^3\mathrm{d}u/\mathrm{d}t$ which coincides with $\xi_{\tau\tau} = x_{tt}\beta^3$ in (26) because the relative u and measured p velocities are opposite in sign, see (15) and (58), thus $x_{tt} = \mathrm{d}u/\mathrm{d}t = -\beta^{-3}\,\mathrm{d}p/\mathrm{d}t$.

The same result can be obtained in a simpler way. Indeed, from the identity in (59) on the left, it follows another identity:

$$p^{-2} + V^{-2} = v^{-2} \equiv u^{-2}, \tag{72}$$

which is valid also for variable velocities p, v, u at $V = \text{const}$. Differentiating (72), we get $a(t) = \mathrm{d}p/\mathrm{d}t = -\gamma^3(p)\mathrm{d}u/\mathrm{d}t$ again. Since from (31) we have

$$\varepsilon G(x) = \mu\beta^3 x_{tt} = \varepsilon x_{tt}, \quad \text{thus,}\ G(x) = x_{tt}, \tag{73}$$

replacing in (71) $\mathrm{d}u/\mathrm{d}t = x_{tt}$ by $G(u)$, we have

$$p\mathrm{d}p/\mathrm{d}t = u\gamma^4 G(u) = -p\gamma^{-1}\gamma^4 G(u), \quad \mathrm{d}p/\mathrm{d}t = a(t), \tag{74}$$

or

$$G(u) = -\gamma^{-3}(p)a(t), \tag{75}$$

where $a(t)$ is computed by (62). Now we have to explain the choice of arguments in the intensity of gravitational field $G(.)$ in (31) and in (73)–(75). In (31), the intensity is denoted by $G(x)$since the coordinate system employed by Einstein for the Maxwell–Hertz equations (18)–(20) in Section 4, and (29)–(30) in Section 6, remains *the same* in (31) if the intensity of gravitation is substituted in place of the electromagnetic field. In general, the gravitation cannot be referred to a *fixed* system of coordinates. Indeed, gravitating masses in the universe are in constant motion. For example, the Earth appears to be in at least three different motions: around its own axis, around the Sun, and with the Sun in the Galaxy. If we choose any *fixed* system of coordinates, then at a *fixed* moment of time the intensity of gravitation is well defined at every point. However, at the next moment, the configuration of masses is changed, and those changes cannot be accounted due to the multitude and diversity of motions of different gravitating masses. For this reason, it seems highly problematic to try to include the gravitation created by moving masses into a fixed set of mathematical formulae (usually called the invariants) that may correspond to a certain choice of the coordinate system. To avoid such difficulties, we changed the argument x in (31) and (73) for u in (74) and (75), which defines the intensity $G(u)$ in the direction of the vector u, at a point where the acceleration $a(t)$ is measured while the body is moving at the speed $|u|$. This point of view is supported by the fact that the gravitation and other fields of forces act on velocities and *not* on the coordinates of a moving body. The result is a procedure for combined identification of the motion and the forces, on the basis of the measurements and computation of velocities and accelerations. To measure the intensity of gravitational field, an asteroid or a dummy spacecraft can be used for reflection of signals sent from a point x_0 of observation, and its location as well as the coordinates of the moving body are also identified by reflected signals.

Remark 10.2. In "The General Theory of Relativity" [8, p. 61], Einstein wrote: "…the gravitational field influences and even determines the metrical laws of the space–time continuum. If the laws of configuration of ideal rigid bodies are to be expressed geometrically, then in the presence of a gravitational field the geometry is not Euclidean. …The most important point of contact between Gauss's theory of surfaces and the general theory of relativity lies in the metrical properties upon which the concepts of both theories, in the main, are based". With due respect to such attempts to convert a variable field of forces into some metrical properties of the relativistic space–time, we regard this as a substitution of one subject by another, not a simpler one, which is difficult to treat and even to measure. For this reason, a method is proposed to directly measure the combined action of unknown fields of forces in some direction of interest, and then to reconstruct the motion in this direction under the measured intensity of the actually existing fields, without reformulating it in terms of a specific space–time geometry.

11. Conclusions

In this paper, some interesting problems of general relativity are considered regarding the information transmittal, gravitation, the notion of mass, and the increased tensions of the electromagnetic field at high velocities in particle accelerators and colliders.

1. It is demonstrated that the hypothesis about the equality of the inert and gravitational masses of a material point is correct only for points *at rest* in a *still* system of coordinates. For a point moving in a gravitational field, its gravitational mass $\varepsilon = \mu\beta^3$, where μ is its inert mass in the second Newton's law of motion and β is the calibration factor in Einstein's relativistic transformations. If velocity $v = 0$ or the speed of information transmittal signals $V = \infty$ (Newtonian far-action), then $\beta = 1$, so that $\varepsilon = \mu$, which is seen in the rays of light at $V = 300\,000$ km/s, as all bodies are falling from $v = 0$ in the gravitational field with the same acceleration. Otherwise, the hypothesis $\varepsilon = \mu$ for a point moving at $v > 0$ with a finite speed of observation signals $V < \infty$ contradicts the principle of relativity for the uniformly accelerated rectilinear motions.
2. In fact, the hypothesis of equality of the gravitational and inert masses $\varepsilon = \mu$ is not required. If $\varepsilon = \mu\beta^3$, then the weight $P = \varepsilon G(x) = \varepsilon x_{tt} = \mu\beta^3 x_{tt} = mg$, the well known formula of elementary physics. If $v = 0$, then $\beta = 1$ and $\varepsilon = \mu$. If $v \to V$, then $\beta \to \infty$, and we have $m = \varepsilon = \mu\beta^3 \to \infty$, so that for a *finite* density $G(x)$ of the gravitational field, the relativistic, thus, *distorted* force of attraction $P = \mu\beta^3 G(x) \to \infty$ as $\beta \to \infty$, an illusory effect sometimes called a black hole.
3. A difference in Einstein's longitudinal $\mu\beta^3$ and transverse $\mu\beta^2$ inert masses appears for the same scalar inert mass μ due to different *observed* accelerations $\xi_{\tau\tau} = x_{tt}\beta^3$ and $\eta_{\tau\tau} = y_{tt}\beta^2$, without affecting the unique scalar Newtonian inert mass μ.
4. The law of preservation of energy in the transformation from the electromagnetic to the *observed* kinetic energy (Einstein) is extended and applied to measure the intensity of gravitational fields, which allows us to identify the force of gravity that accelerates a moving body whereupon this force is being measured by signals reflected from that body. On this basis, a method for relativistic identification of the gravitational and/or electromagnetic fields is developed through the measurements and computation of the actual accelerations along a discretized piece-wise linear trajectory of a moving body.
5. To comply with the nature of observation, the information transmittal signals are incorporated in the Lorentz invariant of the 4D geometry, leading to the local invariants of relativistic dynamics that include gravitation and correspond to realistic physical processes interacting through specific information transmittal signals.
6. The possibility of the relativistic meltdown due to the generation of excessive heat at high velocities of accelerated particles is demonstrated, in order to assure that measures be taken for the safety of physical experiments in particle accelerators and colliders.

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